# Mark Scheme 4726
# June 2007

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| **1** | | Correct formula with correct $r$ | M1 | Allow $r^2 = 2\sin^2 3\theta$ |
| | | Rewrite as $a + b\cos 6\theta$ | M1 | $a, b \neq 0$ |
| | | Integrate their expression correctly | A1√ | From $a + b\cos 6\theta$ |
| | | Get ⅓π | A1 | cao |
| **2** | (i) | Expand to $\sin 2x \cos\frac{1}{4}\pi + \cos 2x \sin\frac{1}{4}\pi$ | B1 | |
| | | Clearly replace $\cos\frac{1}{4}\pi$, $\sin\frac{1}{4}\pi$ to A.G. | B1 | |
| | (ii) | Attempt to expand $\cos 2x$ | M1 | Allow $1 - 2x^2/2$ |
| | | Attempt to expand $\sin 2x$ | M1 | Allow $2x - 2x^3/3$ |
| | | Get $\frac{1}{2}\sqrt{2}\,(1 + 2x - 2x^2 - 4x^3/3)$ | A1 | Four correct unsimplified terms in any order; allow bracket; AEEF |
| | | | | SR Reasonable attempt at $f^{n}(0)$ for $n = 0$ to 3 M1 |
| | | | | Attempt to replace their values in Maclaurin M1 |
| | | | | Get correct answer only A1 |
| **3** | (i) | Express as $A/(x-1) + (Bx+C)/(x^2+9)$ | M1 | Allow $C=0$ here |
| | | Equate $(x^2+9x)$ to $A(x^2+9) + (Bx+C)(x-1)$ | M1√ | May imply above line; on their P.F. |
| | | Sub. for $x$ or equate coeff. | M1 | Must lead to at least 3 coeff.; allow cover-up method for $A$ |
| | | Get A=1, B=0, C=9 | A1 | cao from correct method |
| | (ii) | Get $A\ln(x-1)$ | B1√ | On their $A$ |
| | | Get $C/3\tan^{-1}(x/3)$ | B1√ | On their $C$; condone no constant; ignore any $B \neq 0$ |
| **4** | (i) | Reasonable attempt at product rule | M1 | Two terms seen |
| | | Derive or quote diff. of $\cos^{-1}x$ | M1 | Allow + |
| | | Get $-x^2(1 - x^2)^{-½} + (1 - x^2)^{½} + (1 - x^2)^{-½}$ | A1 | |
| | | Tidy to $2(1 - x^2)^{½}$ | A1 | cao |
| | (ii) | Write down integral from (i) | B1 | On any $k\sqrt{(1-x^2)}$ |
| | | Use limits correctly | M1 | In any reasonable integral |
| | | Tidy to ½π | A1 | |
| | | | SR | Reasonable sub. B1 |
| | | | | Replace for new variable and attempt to integrate (ignore limits) M1 |
| | | | | Clearly get ½π A1 |

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| **5** | (i) | Attempt at parts on $\int 1\,(\ln x)^n\,dx$ | M1 | Two terms seen |
| | | Get $x\,(\ln x)^n - \int n\,(\ln x)^{n-1}\,dx$ | A1 | |
| | | Put in limits correctly in line above | M1 | |
| | | Clearly get A.G. | A1 | ln e =1, ln1 =0 seen or implied |
| | (ii) | Attempt $I_3$ to $I_2$ as $I_3 = e - 3I_2$ | M1 | |
| | | Continue sequence in terms of In | A1 | $I_2 = e-2I_1$ and/or $I_1 = e-I_0$ |
| | | Attempt $I_0$ or $I_1$ | M1 | ($I_0$ =e-1, $I_1$=1) |
| | | Get 6 – 2e | A1 | cao |
| **6** | (i) | Area under graph (= $\int 1/x^2\,dx$, 1 to $n$+1) < Sum of rectangles (from 1 to $n$) | B1 | Sum (total) seen or implied eg diagram; accept areas (of rectangles) |
| | | Area of each rectangle = Width x Height = 1 x $1/x^2$ | B1 | Some evidence of area worked out – seen or implied |
| | (ii) | Indication of new set of rectangles | B1 | |
| | | Similarly, area under graph from 1 to $n$ > sum of areas of rectangles from 2 to $n$ | B1 | Sum (total) seen or implied |
| | | Clear explanation of A.G. | B1 | Diagram; use of left-shift of previous areas |
| | (iii) | Show complete integrations of RHS, using correct, different limits | M1 | Reasonable attempt at $\int x^{-2}\,dx$ |
| | | Correct answer, using limits, to one integral | A1 | |
| | | Add 1 to their second integral to get complete series | M1 | |
| | | Clearly arrive at A.G. | A1 | |
| | (iv) | Get one limit | B1 | Quotable |
| | | Get both 1 and 2 | B1 | Quotable; limits only required |

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| **7** | (i) | Use correct definition of cosh or sinh $x$ | B1 | Seen anywhere in (i) |
| | | Attempt to mult. their cosh/sinh | M1 | |
| | | Correctly mult. out and tidy | A1√ | |
| | | Clearly arrive at A.G. | A1 | Accept $e^{x-y}$ and $e^{y-x}$ |
| | (ii) | Get $\cosh(x-y) = 1$ | M1 | |
| | | Get or imply $(x-y) = 0$ to A.G. | A1 | |
| | (iii) | Use $\cosh^2 x = 9$ or $\sinh^2 x = 8$ | B1 | |
| | | Attempt to solve cosh x = 3 (not –3) or sinh $x = \pm\sqrt{8}$ (allow $+\sqrt{8}$ or $-\sqrt{8}$ only) | M1 | $x = \ln(3 + \sqrt{8})$ from formulae book or from basic cosh definition |
| | | Get at least one $x$ solution correct | A1 | |
| | | Get both solutions correct, $x$ and $y$ | A1 | x, y = $\ln(3 \pm 2\sqrt{2})$; AEEF |
| | | | | SR Attempt tanh = sinh/cosh B1 |
| | | | | Get tanh x = $\pm\sqrt{8}/3$ (+ or -) M1 |
| | | | | Get at least one sol. correct A1 |
| | | | | Get both solutions correct A1 |
| | | | | SR Use exponential definition B1 |
| | | | | Get quadratic in $e^x$ or $e^{2x}$ M1 |
| | | | | Solve for one correct $x$ A1 |
| | | | | Get both solutions, $x$ and $y$ A1 |
| **8** | (i) | $x_2 = 0.1890$ | B1 | |
| | | $x_3 = 0.2087$ | B1√ | From their $x_1$ (or any other correct) |
| | | $x_4 = 0.2050$ | B1√ | Get at least two others correct, |
| | | $x_5 = 0.2057$ | | all to a minimum of 4 d.p. |
| | | $x_6 = 0.2055$ | | |
| | | $x_7 (= x_8) = 0.2056$ (to $x_7$ minimum) | | |
| | | $\alpha = 0.2056$ | B1 | cao; answer may be retrieved despite some errors |
| | (ii) | Attempt to diff. f($x$) | M1 | $k/(2+x)^3$ |
| | | Use $\alpha$ to show $f'(\alpha) \neq 0$ | A1√ | Clearly seen, or explain $k/(2+x)^3 \neq 0$ as $k \neq 0$; allow ± 0.1864 |
| | | | SR | Translate $y=1/x^2$ M1 |
| | | | | State/show $y=1/x^2$ has no TP A1 |
| | (iii) | $\delta_3 = -0.0037$ (allow –0.004) | B1√ | Allow ±, from their $x_4$ and $x_3$ |
| | (iv) | Develop from $\delta_{10} = f'(\alpha)\,\delta_9$ etc. to get $\delta_i$ or quote $\delta_{10} = \delta_3\, f'(\alpha)^7$ | M1 | Or any $\delta_i$ eg use $\delta_9 = x_{10} - x_9$ |
| | | Use their $\delta_i$ and $f'(\alpha)$ | M1 | |
| | | Get 0.000000028 | A1 | Or answer that rounds to ± 0.00000003 |

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| **9** | (i) | Quote $x = a$ | B1 | |
| | | Attempt to divide out | M1 | Allow M1 for y=x here; allow |
| | | | A1 | $(x\text{-}a) + k/(x\text{-}a)$ seen or implied |
| | | Get $y = x - a$ | A1 | Must be equations |
| | (ii) | Attempt at quad. in x (=0) | M1 | |
| | | Use $b^2 - 4ac \geq 0$ for real $x$ | M1 | Allow > |
| | | Get $y^2 + 4a^2 \geq 0$ | A1 | |
| | | State/show their quad. is always >0 | B1 | Allow ≥ |
| | (iii) | | B1√ | Two asymptotes from (i) (need not be labelled) |
| | | | B1 | Both crossing points |
| | | | B1√ | Approaches – correct shape |
| | | | SR | Attempt diff. by quotient/product rule M1 |
| | | | | Get quadratic in $x$ for $dy/dx = 0$ and note $b^2 - 4ac < 0$ A1 |
| | | | | Consider horizontal asymptotes B1 |
| | | | | Fully justify answer B1 |